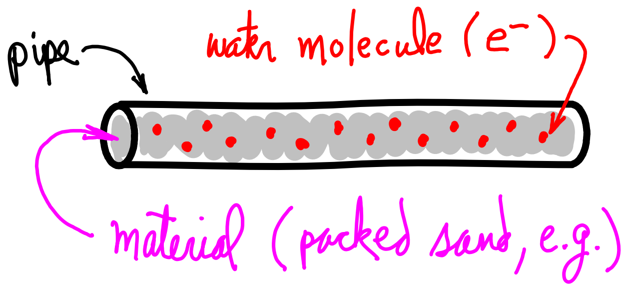
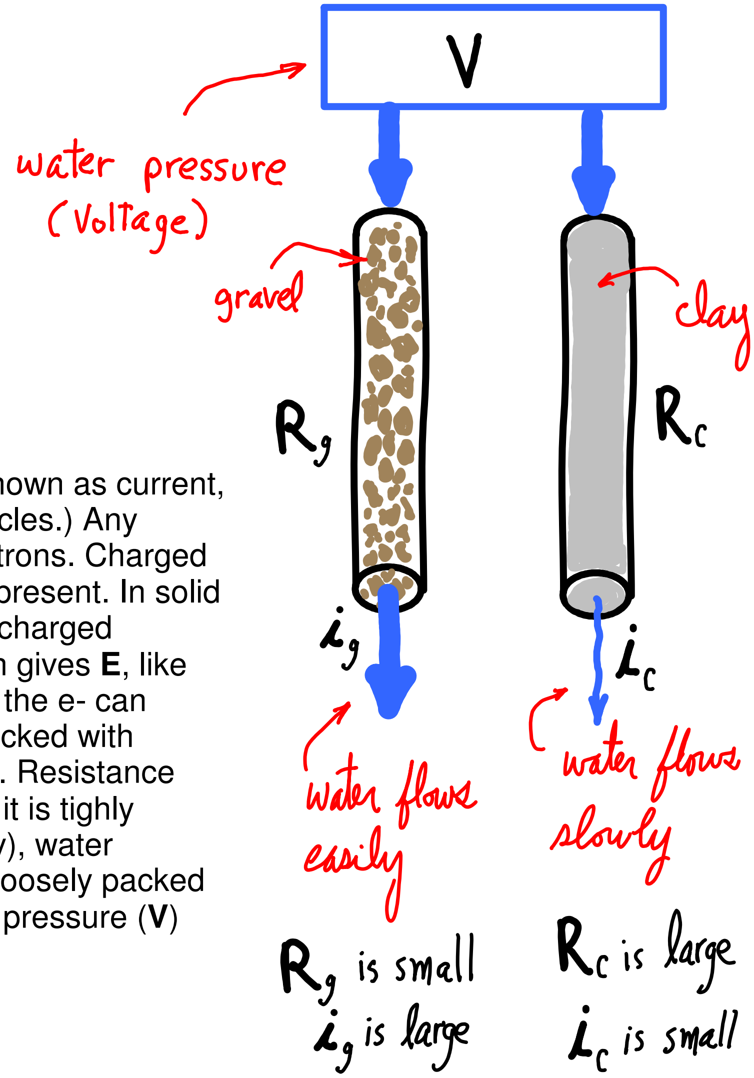
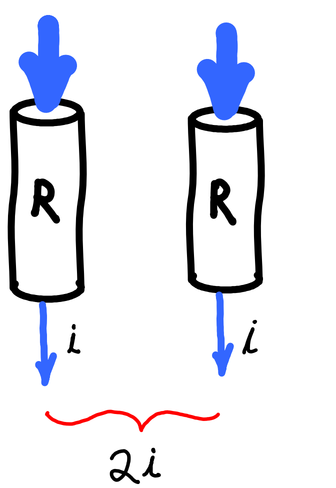
Basic Electricity, The Water Analogy





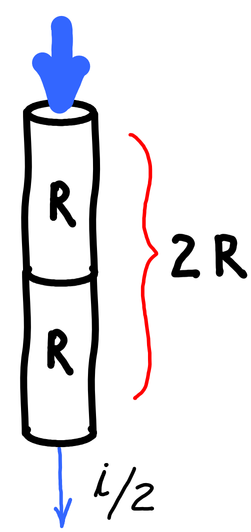
Conductors

Conduction is the movement of electrons (**e-**), also known as current, i. (Conduction can also be by positively charged particles.) Any material conducts, if we pull hard enough on the electrons. Charged things, such as e-, move when an electric field (**E**) is present. In solid materials, the nucleus of an atom contains positively charged protons (**p+**). Protons and e- attract each other, which gives **E**, like gravity. In solid material the p+ are fixed in place, but the e- can move. We can think of the solid material as a pipe packed with something; e.g., sand, and the e- as water molecules. Resistance (**R**) is how tightly packed the material in the pipe is: if it is tighly packed (the material is very fine material such as clay), water molecules have a hard time making it through; if it is loosely packed (large gravel), water drains through easily. The water pressure (**V**) and **R** together determine i.



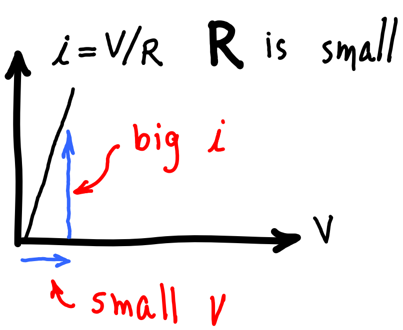
Parallel circuit
Suppose we have two identical pipes side by side, and they both have resistance, **R**, and each has current, i. The current through both is twice the current through one, $2i$.

Series circuit
If we connect them end to end instead, we might expect the total resistance to be **R+R**, and the current to be $(1/2)\, i$.

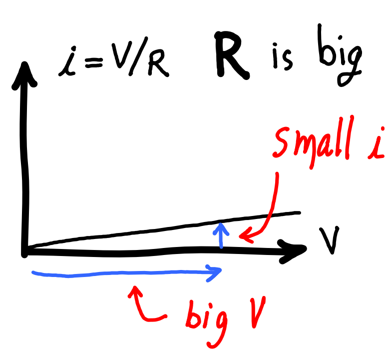


Ohm's Law devices
Different materials and devices have different relationships between i, **R**, and **V**.If the relationship can be expressed as,

$$i\,\mathbf{R} = \mathbf{V}$$



then we call the device an Ohm's Law device. Of course, this is only an approximate model. If **R is very big**, the device **is a non-conducting insulator**. A **big pressure V** only gives **a little flow**. If **R is very small**, the device **is a conductor**. A very **small V** gives a **large flow**.



It takes work to get water pressure. Suppose we have a water tower. We pull the water up. **Pulling the weight w up the tower's height h** is the **work** we do, **w X h**.

We can get that same amount of energy back from the water in the tank. We can drop a bucket of water and use the pull to do work of some sort. That energy is used up in our packed pipe as the **water falls** through the pipe: it **heats the packing** as the water collides with the packing. The **heat escapes** by radiating away.

The downward force on the water is caused by the gravity **field E** acting on the water's mass: the **weight** of the water is **E X mass**. On the moon, the same mass of water weighs less because the moon's gravity field pulls less than earth's. You can jump high easily on the moon, for instance. **Smaller E** would mean it takes **less energy** to move the water: **less pressure** in the pipe, and **less flow**, and **less heat**.

Our model of an electrical **voltage source** is a tower and **very large pipe without packing**. It supplies water that flows through our packed pipe, and an **energetic process pumps** water back up. The pressure at the pump inlet is **V-** and the pressure at the tank end is **V+**. The **pressure difference V** drives water through the **packed pipe**.

Voltage Source = Pump + Tank + Big Pipe

Device/Circuit = Packed Pipe

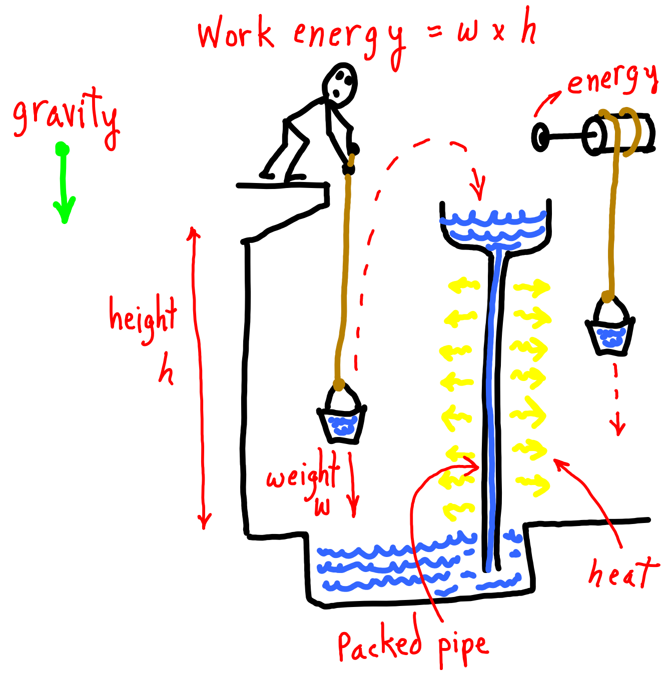
The **energy lost** in the packed pipe by the water that flows through it is the same it took to pump the water into the tank:

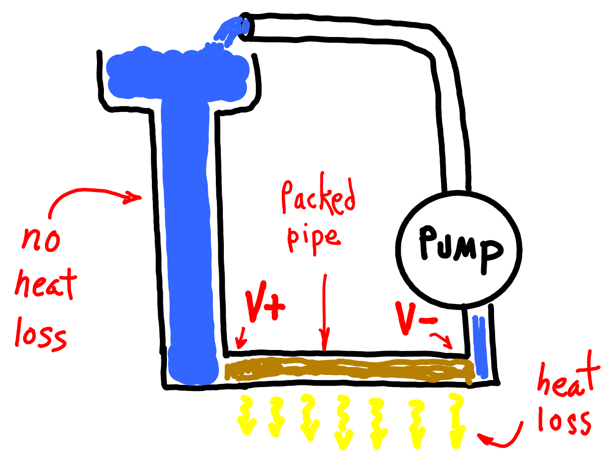
energy = weight X h
weight = mass X **g** (**g** is gravitational acceleration)
mass = volume X density (let density = 1)
volume = Area X **h**

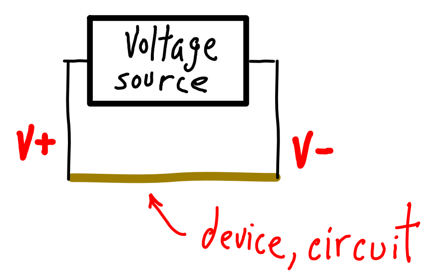
energy = (Area X h X 1) X **g** X **h**

Here, we are assuming the volume of water that flowed is equal to the volume of the big pipe whose cross sectional area is Area. Because the big pipe is h tall, its volume is Area X h. Suppose we want to see **how much energy** an amount of water of **mass m** loses. We first find the energy lost per unit mass by dividing the above by the mass (Area X h X 1):

energy-per-unit-mass = **g** X **h**







The **energy from mass m** is then:

energy-m = **m** X **g** X **h**

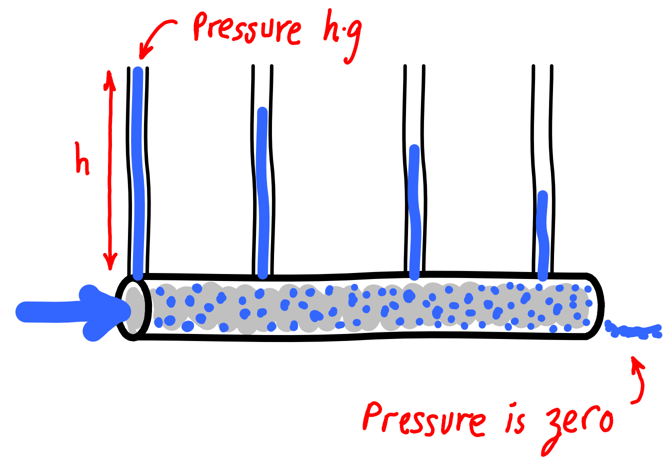
Define **V** (short for voltage-across-the-device) as (**g** X **h**):

V = (**g** X **h**)

The energy for mass **m** is then,

energy-m = **m** X **V**

A packed pipe with water flowing through it has more pressure on the inlet side than the outlet side. (If it were the other way around, the flow would go backward.) The pressure drops along the pipe. At the inlet side, the pressure is just the total weight of the water in the big pipe pressing down divided by its area:



pressure-h = **(Area X h X 1)** X **g** / **Area**

= **g** X **h**

= **V**

So, the voltage **V is** the same as the **water pressure** supplied by the source. **Exit pressure is zero** because the **pump is pulling** the water from that end of the pipe.

Suppose **k** units of water flow per second. The **power loss in the device** is,

Power = **energy-per-unit-mass** X (**k**/sec)

= **V** X (**k**/sec)

Current is i and is equal to (**k**/sec):

Power = **V** X i

Electrons and water molecules are equivalent. They just differ in their respective fields (**E** and **g**) and the properties those fields affect (**charge** and **mass**). **Power loss is heat** (mostly). Note that we have used **E** and **g** interchangeably, and applied electrical terminology to water.

Suppose our packed pipes can be modeled by Ohm's Law. **Power loss** is then **proportional to the square** of **V**. It can also be expressed as proportional to the square of i. (Both are shown at right.)

$$\text{power} = iV \qquad V = iR$$
$$= i(iR) = i^2 R$$

$$\text{power} = iV \qquad i = V/R$$
$$= (V/R)V = V^2/R$$

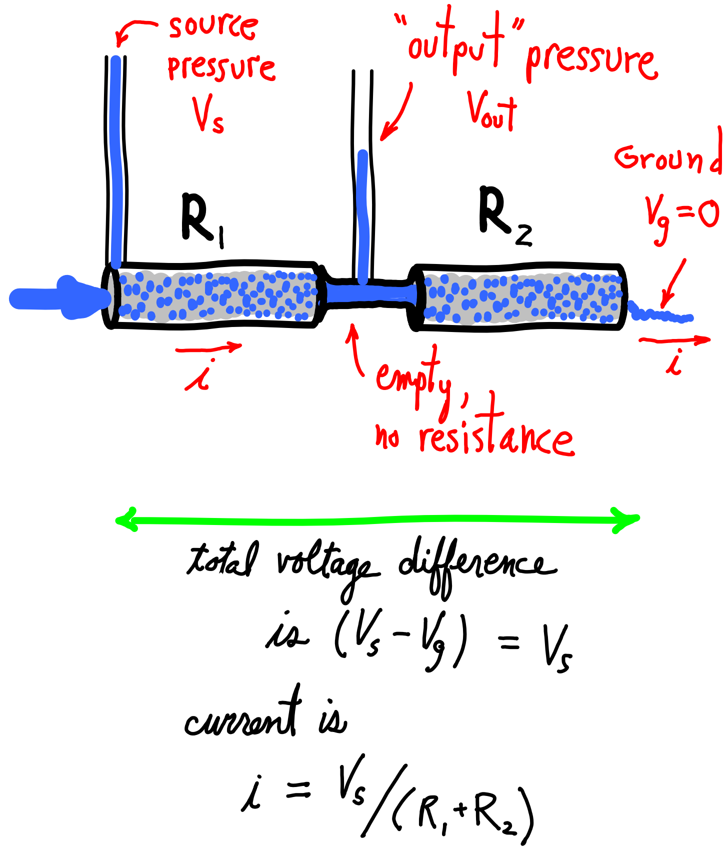
Note for unchanging **V**, that as resistance **R goes to 0**, the **power loss goes to infinity**. This is a short circuit. Before power loss actually goes to infinity, the heat will melt or vaporize the device. Of course, as **R goes to infinity**, nothing will flow, and **no power is lost**.

$$\lim_{R \to 0} \text{power} = \lim_{R \to 0} V^2/R$$

Voltage Divider

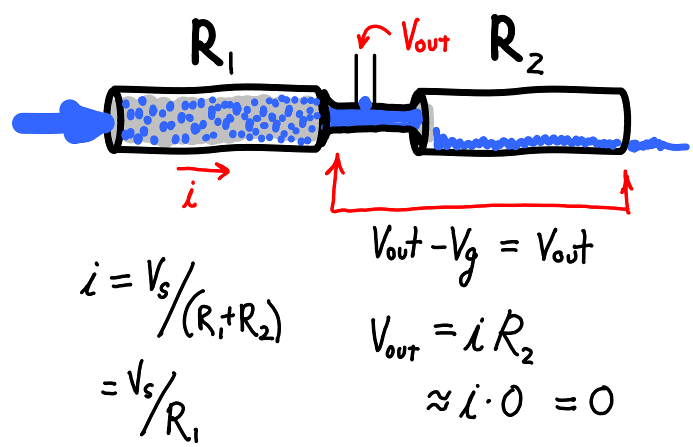
At right are two devices connected in series: Between them is a section of empty pipe whose resistance is relatively close to zero (wire).

The pressure (voltage) across **device1** is the source voltage $\mathbf{V}_s$ minus the "output" voltage $\mathbf{V}_{out}$. The current exiting **device2** has pressure $\mathbf{V}_g = 0$. So, the voltage across **device2** is $\mathbf{V}_{out}$.



The "output" of this system $\mathbf{V}_{out}$ depends on the two resistances, $\mathbf{R}_1$ and $\mathbf{R}_2$. The current i is the same through both resistors.

Suppose $\mathbf{R}_2$ **is nearly 0** (a resistance-less wire). The total voltage difference over both resistors is $(\mathbf{V}_s - \mathbf{V}_g) = \mathbf{V}_s$. The output voltage is the voltage difference across $\mathbf{R}_2$. Because $\mathbf{R}_2$ is about 0, it has no packing, water passes through easily, 0 volts is almost all that is needed to move water through it. That is to say, no water pressure can build up on the inlet side of $\mathbf{R}_2$ because when it starts to build up, water flows through before any pressure can build up. The current i is completely determined by $\mathbf{R}_1$.



In the other extreme, suppose $\mathbf{R}_2$ is **nearly infinite** (an open circuit or switch). No matter how much pressure there is, almost no current flows.

$$i = (\mathbf{V}_s - \mathbf{V}_g) / (\mathbf{R}_1 + \mathbf{R}_2) = \mathbf{V}_s / (\mathbf{R}_1 + \mathbf{R}_2) \sim \mathbf{0}$$

Pressure will build up as flow exits $\mathbf{R}_1$ and gets stopped by $\mathbf{R}_2$. Water would flow through $\mathbf{R}_2$ if the pressure at one end were different from the pressure at the other end. But, no current flows. So, the pressure at both ends must be the same. That is, $\mathbf{V}_{out}$ is the same as $\mathbf{V}_s$.

$$(\mathbf{V}_s - \mathbf{V}_{out}) = i\,\mathbf{R}_1 \sim 0\,\mathbf{R}_1 = \mathbf{0}$$

$$\mathbf{V}_s \sim \mathbf{V}_{out}$$